MATH 281BC – Qualifying Exam – Fall 2019

Use the notation defined in the reference sheet as much as possible; otherwise, define any symbol you use. Name any result you use from the reference sheet. Be concise and clear. Justify all your answers. All tests are considered at some prescribed level α unless otherwise specified.

Problem 1. Let $X_1, \dots, X_n$ be iid $\mathcal{N}(a, \sigma^2)$, and independently, let $Y_1, \dots, Y_n$ be iid $\mathcal{N}(b, \sigma^2)$.

A. Assume σ^2 is known. Consider testing $a \geq b$ versus $a < b$. Derive a UMP test at level α. [There is a point mass prior that is least favorable.]

B. Assume σ^2 is known. Consider testing $a = b$ versus $a \neq b$. Derive a UMPU test at level α.

C. Assume σ^2 is *unknown*. Consider testing $a = b$ versus $a \neq b$. Is there a UMPU test at level α?

Problem 2. Consider a setting where $\theta \sim \mathcal{N}(0, \tau^2)$ and, given θ, $X_1, \ldots, X_n \overset{\text{iid}}{\sim} \mathcal{N}(\theta, \sigma^2)$, with τ^2 and σ^2 both known (for now). We consider the problem of estimating θ under square error loss.

A. Derive the Bayes estimator.

B. Derive the Bayes risk.

C. Prove that the sample mean (denoted $\bar{X}$) is minimax.

D. Is $\bar{X}$ still minimax when σ^2 is unknown? What if we know that $\sigma^2 \leq 1$?

E. Is $\bar{X}$ still minimax when in addition we know that $\theta \geq 0$?

F. Prove that $\bar{X}$ is admissible.

G. Is $\bar{X} + b$ admissible when $b \neq 0$?

H. Derive the empirical Bayes estimator for θ.