Student full name:

MATH 281BC – Qualifying Exam – Spring 2018

Keep the reference sheet and use the notation defined there and in general notation used in lecture. Otherwise, define any symbol and name any result you use from the reference sheet. Be concise and clear.

Problem 1.

A. Show that when $\bar{G}$ is transitive over Ω, the risk function of any equivariant estimator is constant.

B. Suppose ρ is convex and even function and that Z is a random variable with a symmetric distribution about 0. Show that $\phi : a \mapsto \mathbb{E}[\rho(Z - a)]$ is convex and even.

C. In a location-scale setting, show that a function q is invariant if and only if $q(x_1, \dots, x_n) = u(z_1, \dots, z_{n-2})$ where $z_j = (x_j - x_n)/(x_{n-1} - x_n)$. Deduce that an estimator δ is equivariant in this setting if and only if $\delta(x_1, \dots, x_n) = \delta_0(x_1, \dots, x_n) - u(z_1, \dots, z_{n-2})$, where δ_0 is a given equivariant statistic and u is arbitrary. Give an example of δ_0.

D. Prove M1 from the summary sheet.

E. Prove M2 from the summary sheet.

F. Consider testing $X \sim f_0$ versus $X \sim f_1$, where f_0 and f_1 are densities with respect to some measure. Assume that $f_1(X)/f_0(X)$ has a continuous distribution when $X \sim f_0$. Give a most powerful test at level α. Is it unique?

G. Define what it means for a test to be unbiased level α for testing $\theta \in \Omega_H$ versus $\theta \in \Omega_K$.

Problem 2. Consider $X_1, \ldots, X_n$ iid uniform in $[a - b/2, a + b/2]$, where $a \in \mathbb{R}$ and $b > 0$ are both unknown.

A. Show that this is a location-scale family.

B. Show that $X_{(1)} = \min_i X_i$ and $X_{(n)} = \max_i X_i$ are jointly sufficient.

C. Derive the MLE for a. (If it is not unique, make a natural choice if possible.) Is it equivariant in some way?

D. With square error loss, derive the MRE for a.

Problem 3. Let $X_1, \dots, X_n$ be iid $\mathcal{N}(a, \sigma^2)$, and independently, let $Y_1, \dots, Y_n$ be iid $\mathcal{N}(b, \sigma^2)$.

A. Assume σ^2 is known. Consider testing $a \geq b$ versus $a < b$. Derive a UMP test at level α. [There is a point mass prior that is least favorable.]

B. Assume σ^2 is known. Consider testing $a = b$ versus $a \neq b$. Derive a UMPU test at level α.

C. Assume σ^2 is *unknown*. Consider testing $a = b$ versus $a \neq b$. Is there a UMPU test at level α?