ALGEBRA QUALIFYING EXAM

June 1. 2008

Do All Problems

(1) Let G be a finite group and H a proper subgroup. Show that G is not the set theoretic union of the conjugates of H.

(2) Classify all groups with 99 elements.

(3) Show that if G is a group of order p^n, where p is a prime, and N is a normal subgroup, then N intersects the center of G nontrivially.

(4) Suppose there exists an intermediate field, L, of the Galois extension F/E, of degree 2 over E. What can we say about $\mathrm{Gal}(F/E)$?

(5) Let $p(x)$ be a polynomial over Q with Galois group $Z_4 \oplus Z_4$. What can be said about the solvability of $p(x)$ by radicals? What if the Galois group is S_5?

(6) If K is a subfield of L, show that ring S of all polynomials over L with constant coefficient in K is noetherian if and only if L is finite dimensional over K.

(7) Let A be a simple ring with identity element. Show that if A has a minimal right ideal, then A satisfies the minimum condition for right ideals.

(8) Show that the center of a simple ring with identity element is a field.

(9) Give an example of a ring with the right minimum condition, but not the left. Can you find such an example with no nonzero nilpotent ideals? Why?