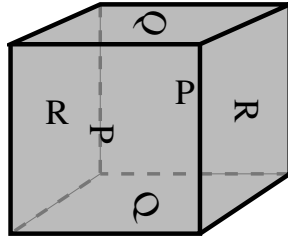


19. Summer 2015

1. Let X be the space obtained by gluing opposite pairs of faces of a standard cube I^3 via 90 degree rotations, as shown. Compute the homology $H_*(X; \mathbb{Z})$.



2. Prove that there is no compact 4-manifold M (with or without boundary) which is homotopy-equivalent to the suspension $\Sigma\mathbb{R}P^3$.
3. Prove that any map $\mathbb{R}P^2 \rightarrow T^2$ must be null-homotopic.
4. Let X_n be a space whose homology groups are given by $H_k(X_n; \mathbb{Z}) \cong \mathbb{Z}/k\mathbb{Z}$ for $0 \leq k \leq n$ and which vanish for $k > n$. Compute the homology $H_*(X_3 \times X_5; \mathbb{Z})$.
5. Let Σ_3 be the closed orientable surface of genus 3. Suppose \mathbb{Z}_3 acts on Σ_3 ; show that there must be at least two fixed points.
6. Let M be a compact 3-manifold-with-boundary such that $H_1(M; \mathbb{Z})$ contains a torsion element (i.e. element of finite order). Prove that M cannot be embedded as a submanifold of S^3 . (Hint: if it could, we could decompose $S^3 = M \cup_{\Sigma} N$ as a union of two compact 3-manifolds, glued along their common boundary surface Σ .)
7. Let $q : S^3 \rightarrow \mathbb{R}P^3$ be the usual quotient map which identifies antipodal points. It can be used to attach a 4-ball to $\mathbb{R}P^3$, forming a space $X = \mathbb{R}P^3 \cup_q B^4$. Compute $\pi_4(X)$.
8. Suppose X is a path-connected CW-complex with $\pi_1(X) \cong \mathbb{Z}^2$ and $\pi_{\geq 2}(X) = 0$. Show that X is homotopy-equivalent to $S^1 \times S^1$. Use this to show that the fundamental group of a closed orientable surface Σ_g of genus $g \geq 2$ cannot contain a subgroup isomorphic to \mathbb{Z}^2 .